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# Understanding Hurst Exponent and Equilibrium Fractal Wave Index for Financial Trading

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## Overview

Financial market is one of the most interesting topics in science. The fractal nature of the financial market was studied more than decades in both academic and industrial research. Many investment banks and fund management firms spend a considerable amount of time and efforts to reveal the fractal properties of the financial market so they can use such a knowledge for their trading and investment decision. Since fractal geometry in the financial market is complex, we need scientific tools to study the structure and the behaviour of the financial market. If we understand the structure and the behaviour of the financial market, we can create better trading strategies for sure. In this article, we will help you to understand two important fractal based scientific tools including Hurst Exponent and Equilibrium Fractal wave index. We explain these two tools in a simple language for the example of financial trading.

## 1. Fractal Nature in the Financial Market

The term fractal was used for the first time by Benoit Mandelbrot (20 November 1924 – 14 October 2010). This is how he defined fractals: “Fractals are objects, whether mathematical, created by nature or by man, that are called irregular, rough, porous or fragmented and which possess these properties at any scale. That is to say they have the same shape, whether seen from close or from far.” This is a general description of the fractals from the father of fractals. At the most plain language, the fractal is the repeating geometry. For example, in Figure 1-1, a triangle is kept repeating to form larger triangles. How big or small we zoom out or zoom in, we can only see the identical triangle everywhere. When the pattern or structure is composed of regular shape as shown in Figure 1-1, we call such a pattern as the strict self-similarity.

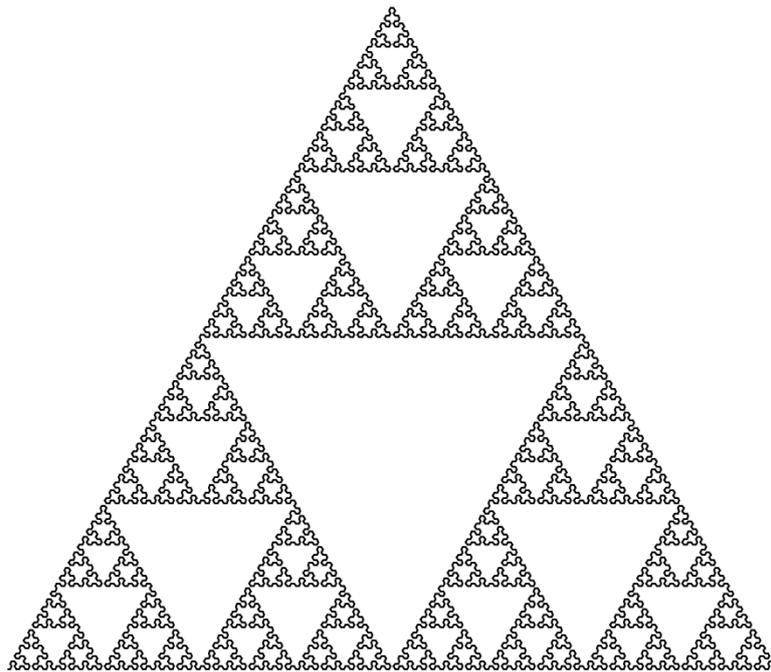


Figure 1-1: Example fractal geometry with strict self-similarity.

Fractal geometry can be found in nature including trees, leaves, mountain edges, coastline, etc. The financial market has also strong fractal nature in it. Since the price of financial instruments is drawn in time and price space, the fractal in the financial market comes in waveform over the time. However, we are not talking about the typical cyclic wave as in the sine or cosine wave, which can be defined with a definite cycle period. In the financial market, we are talking about the repeating geometry or patterns over the time without definite cycle period. Another important fractal characteristic of the financial market is a loose self-similarity in contrast to the strict self-similarity in Figure 1-1. Loose self-similarity means that the financial market is composed of slightly different variation of the regular shape (Figure 1-2). Therefore, to understand the financial market, we need some tools to visualize its structure. If we understand the fractal nature of the financial market, we can definitely improve our trading performance. Many investment banks and fund management firms do spend considerable amount of efforts and time to reveal the fractal properties of the financial market. They use such a knowledge for their trading and investment decision. From the next chapter, we introduce few important scientific tools to reveal the market structure and behaviour of the financial market for your trading.



Figure 1-2: Loose self-similarity of the financial market.

## 2. Hurst Exponent for Financial Trading

The name “Hurst exponent” or “Hurst coefficient” was derived from Harold Edwin Hurst (1880-1978), the British hydrologist. Among the scientists, Hurst exponent is typically used to measure the predictability of time series. In fact, Hurst exponent is theoretically tied to the Fractal dimension index coined by Mandelbrot in 1975. Therefore, when we explain Hurst exponent, we can not avoid to mention about the Fractal Dimension index. The relationship between Hurst exponent and Fractal dimension index is like this:

$$\text{Fractal dimension index (D)} = 2 - \text{Hurst exponent (H)}.$$

Even if we had a definite mathematical relationship between these two, we should interpret them independently. For example, Fractal dimension index can range from one to two. This value corresponding to the typical geometric dimension we know. For example, everyone

knows that one dimension indicates a straight line whereas the two dimension indicates an area. Three dimension is a volume. Of course, for some big science fiction fans, four dimension might be an interesting topic. Now we know that the fractal dimension index can range from 1 to 2. What does 1.5 dimension means? Fractal dimension index 1.5 is simply the filling capacity of the geometric pattern. If the geometric patterns are highly wiggly and then can fill more space than a straight line, the geometric patterns will have higher fractal dimension index. If the geometric pattern is simple, then the pattern will have lower fractal dimension index close to one (i.e. straight line). For the financial market, the fractal dimension index can range somewhere between 1.36 and 1.52. You can imagine how complex they are. It is important to note that the fractal dimension index is not a unique descriptor of shape. Therefore, the number does not tell how the shape of the fractal geometry.

Hurst exponent can range from 0.0 and 1.0. Unlike the fractal dimension index, Hurst exponent tell us the predictability of the financial market. For example, if the Hurst exponent is close to 0.5, this indicates the financial market is random. If the Hurst exponent is close to 0.0 or 1.0, then it indicates that the financial market is highly predictable. The best-known approach using the Hurst exponent for the financial trading is to classify the financial market data into momentum (i.e. trending) and mean reversion (i.e. ranging) characteristics. For example, if Hurst exponent of the financial market is greater than 0.5, then we can assume that the financial market have a tendency for trending. If Hurst exponent is less than 0.5, we can assume that the financial market have a tendency for ranging. Hurst exponent is generally calculated over the entire data. It is used as a metric to describe the characteristic of the financial market. However, there are some traders using Hurst exponent like a technical indicator by calculating them for short period. When you calculate Hurst exponent over short period, you might run the risk of incorrect range analysis (Figure 2-1). For example, it is well known that with small data set, the estimated standard deviation can be far off from the true standard deviation of the population. However, at the same time, if you are using overly long period to calculate Hurst exponent, you will get the lagging signals (Figure 2-2). If you are using Hurst exponent for reasonably long calculating period, then Hurst exponent will not alternate between trending ( $> 0.5$ ) and ranging region ( $< 0.5$ ) but the value will stay only one side (Figure 2-3). In Figure 3, Hurst exponent stayed over 0.57 always when we have the

calculating period 3000 for EURUSD H1 timeframe. It is also important to note that Hurst exponent does not tell you the direction of the market.



Figure 2-1: Hurst Exponent indicator with period 30 on EURUSD H1 timeframe. The green dotted line is at 0.5.



Figure 2-2: Hurst Exponent indicator with period 100 on EURUSD H1 timeframe. The green dotted line is at 0.5.

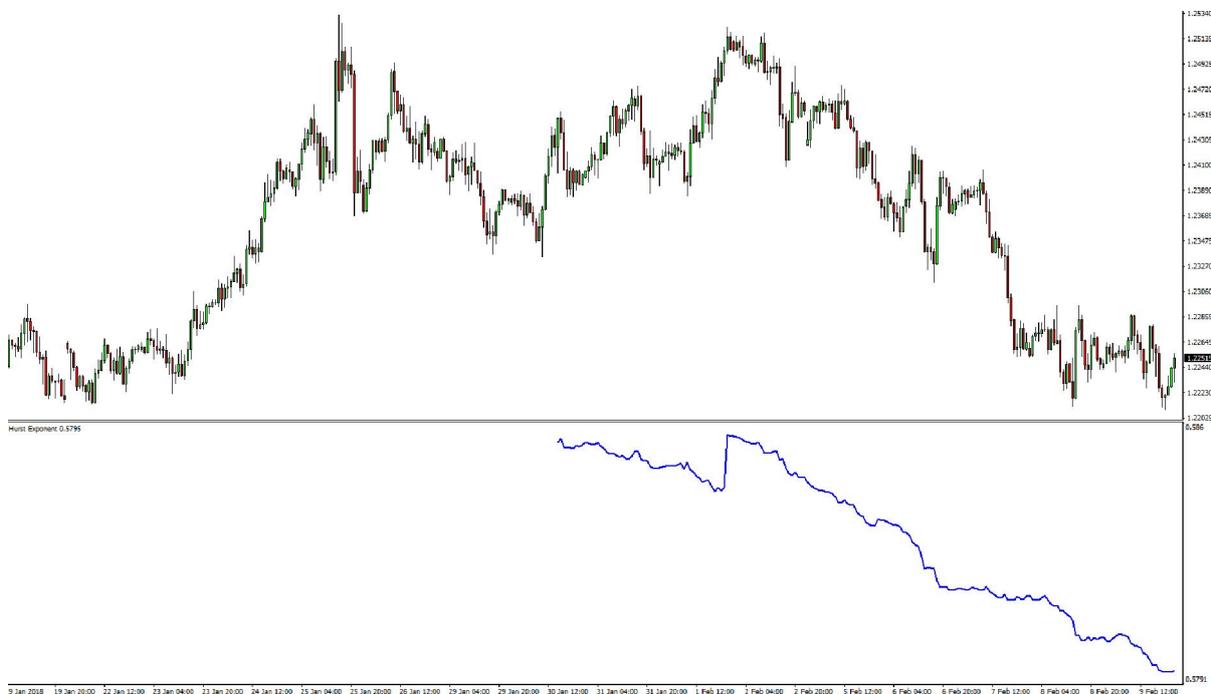


Figure 2-3: Hurst Exponent indicator with period 3000 on EURUSD H1 timeframe. Hurst exponent value is always greater than 0.57.

### 3. Equilibrium Fractal Wave Index for Financial Trading

The Equilibrium fractal wave index was first introduced in the Book: Financial trading with Five Regularities of Nature: Scientific Guide to Price Action and Pattern Trading (2017). If Hurst exponent was created to extract insight for the overall data of the financial market, the Equilibrium fractal wave index was created to extract insight for the fractal geometry in the loose self-similarity system like the financial market. In the Equilibrium fractal wave index, the building block of the fractal geometry is assumed as the simple triangular waveform called equilibrium fractal wave. Remember that in the strict self-similarity system, the fractal geometry is composed of infinite number of regular shape as in Koch Curve and Sierpinski Triangle (Figure 1-1). In the loose self-similarity structure, the fractal geometry is composed of infinite number of slightly different version of the regular shape. Likewise, many different variation of the triangular shape shown in in Figure 3-1 can become the equilibrium fractal wave in the financial market. The variation of shape in the equilibrium fractal wave can be expressed as the Shape ratio of latest price move to previous price move at the two swing points (i.e. the shape ratio =  $Y_2/Y_1$ ). Figure 3-2 and 3-3 show the example of identical shape and non-identical shape of equilibrium fractal wave. Since the financial market is the complex system with loose self-similarity, the financial market is composed of infinite number of some identical and some non-identical shape of equilibrium fractal waves as shown in Figure 3-4. The Equilibrium fractal wave index simply tells you how often the identical shape of equilibrium fractal wave is repeating in the financial market. To help you understand further, the mathematical equation for the Equilibrium Fractal Wave index is shown below:

Equilibrium fractal wave index = number of the particular shape of equilibrium fractal wave / number of peaks and troughs in the price series.

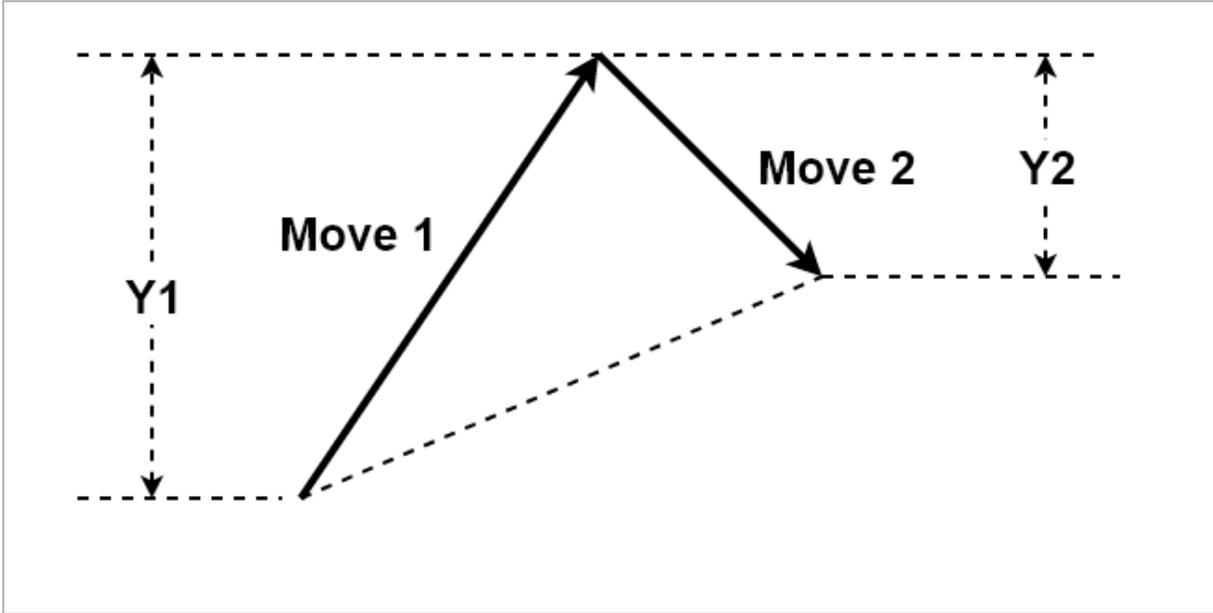


Figure 3-1: Structure of one equilibrium fractal wave. It is made up from two price movements (i.e. two swings).

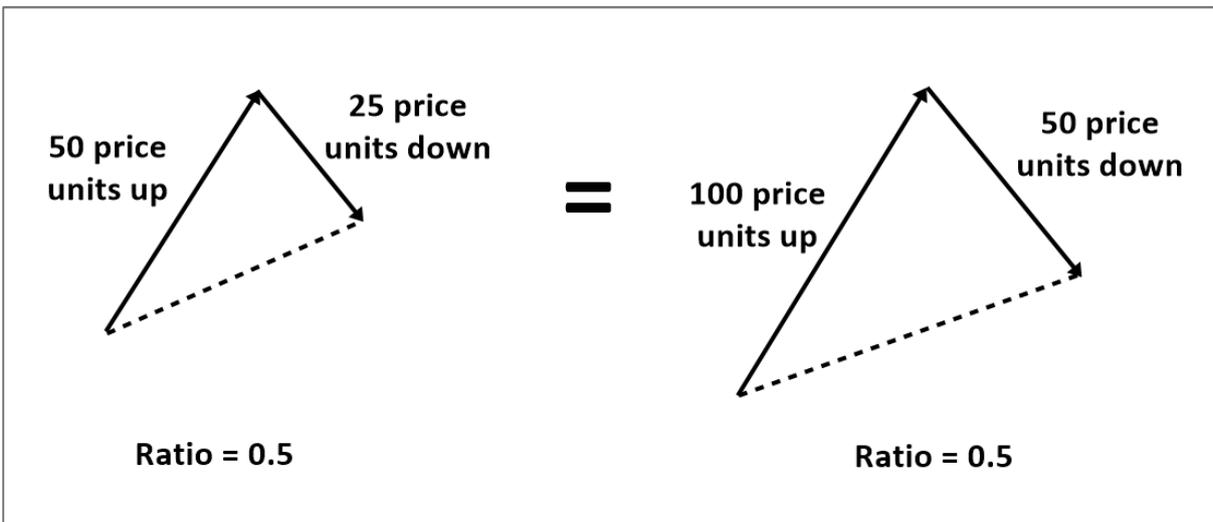


Figure 3-2: An example of two identical equilibrium fractal waves in their shape.

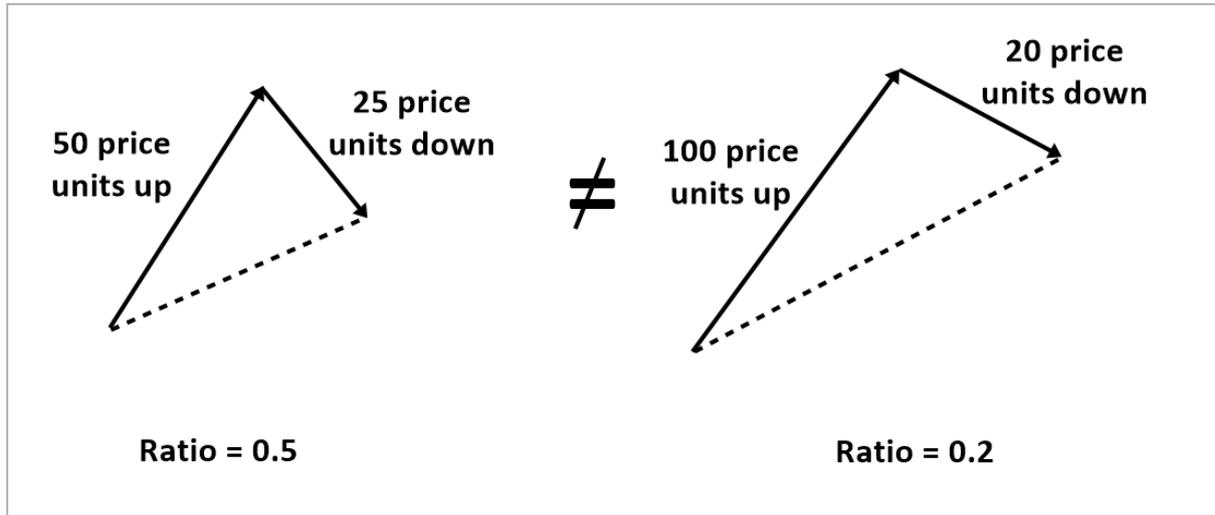


Figure 3-3: An example of non-identical equilibrium fractal waves in their shape.

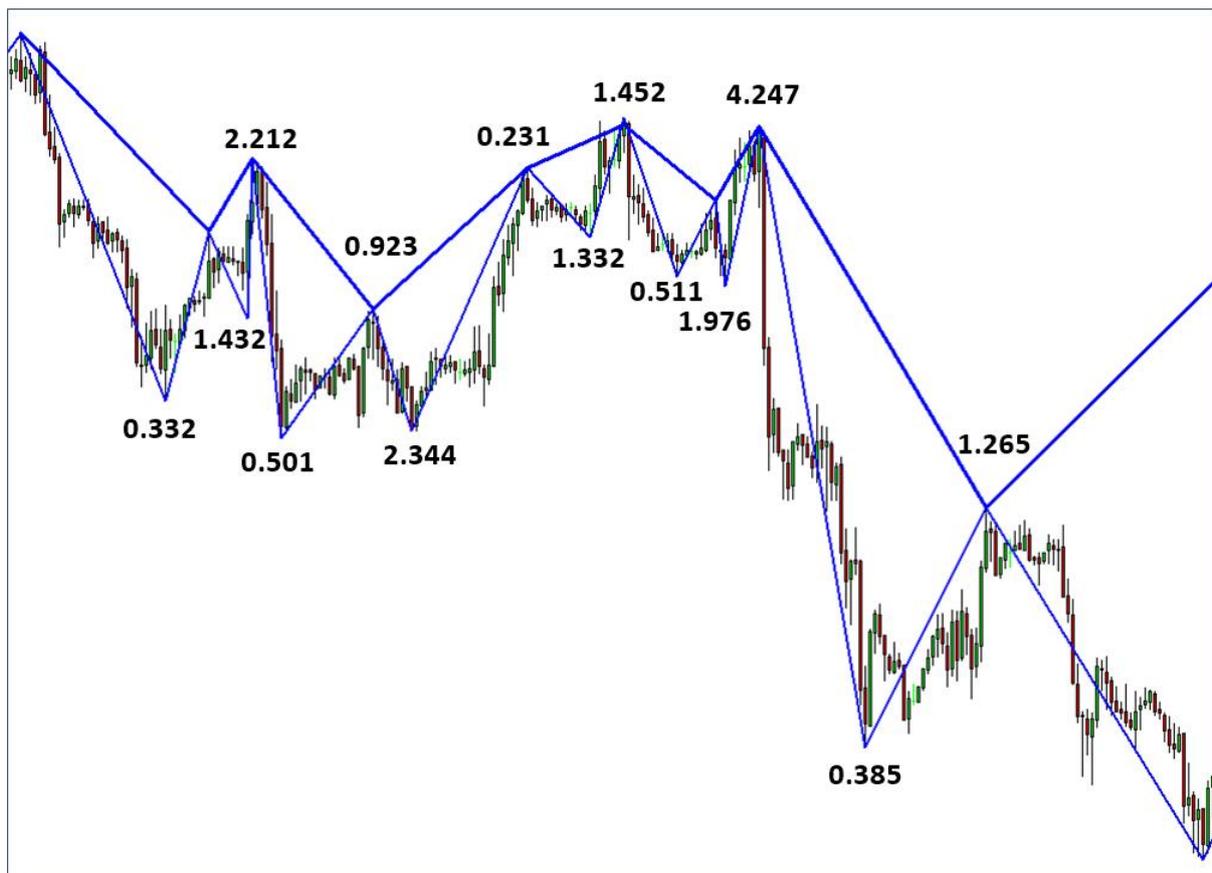


Figure 3-4: Financial market with loose self-similarity. The shape ratio ( $Y_2/Y_1$ ) corresponds to each equilibrium fractal wave.

So how to use Equilibrium fractal wave index for financial trading? If the Hurst exponent tells you the predictability of the financial market, then the Equilibrium fractal wave index can reveal the internal structure of the financial market. For example, Table 3-1 shows the international structure of EURUSD for around 12 years of history data. We can tell how the six different variation of equilibrium fractal waves exist in EURUSD in different proportion. Some variation of equilibrium fractal wave appears more frequently than the other shape ratios. For example, the shape ratio 0.618 (i.e. the golden ratio) and 0.850 appears more frequently than the other shape ratios in EURUSD. The higher the Equilibrium fractal wave index means that the shape ratio indicates reliable trading opportunity whereas the lower the Equilibrium fractal wave means that they are not so significant to trade. With Equilibrium fractal wave index, you can also cross compare the internal structure of different financial instruments. Table 3-2 shows how GBPUSD is composed of these six variation of equilibrium fractal waves. You can tell the composition is not similar to the case of EURUSD (Table 3-1). This simply tells you that each financial instrument have their own behaviour. In addition, with Equilibrium fractal wave index, we can cross-compare the composition for multiple of financial instruments. For example, in Table 3-3, we cross compared the composition of the shape ratio 0.618 for 10 different currency pairs. You can tell that the shape ratio of 0.618 take up the higher proportion in some currency pairs whereas it is not so significant in other currency pairs. For example, the appearance of shape ratio in USDJPY is roughly 25% more than the appearance of the shape ratio in AUDNZD (Table 3-3). This indicates that you will be better off to trade with USDJPY than AUDNZD if your trading strategy involves using the golden ratio 0.618.

Shape Ratio	Start	End	Number of Equilibrium Fractal Wave	Number of Peaks and troughs	EFW Index
0.618	2006 09 20	2018 01 20	108	321	33.6%
0.382	2006 09 20	2018 01 20	99	321	30.8%
0.500	2006 09 20	2018 01 20	102	321	31.8%
0.300	2006 09 20	2018 01 20	65	321	20.2%
0.450	2006 09 20	2018 01 20	101	321	31.5%
0.850	2006 09 20	2018 01 20	138	321	43.0%
Sum			613	321	190.97%
Average			102.17	321	31.83%
Stdev			23.28	0.00	N/A

Table 3-1: Internal structure of EURUSD D1 timeframe from 2006 09 20 to 2018 01 20 with six different shape ratios.

Shape Ratio	Start	End	Number of Equilibrium Fractal Wave	Number of Peaks and troughs	EFW Index
0.618	2007 01 04	2018 01 20	116	339	34.2%
0.382	2007 01 04	2018 01 20	95	339	28.0%
0.500	2007 01 04	2018 01 20	124	339	36.6%
0.300	2007 01 04	2018 01 20	62	339	18.3%
0.450	2007 01 04	2018 01 20	114	339	33.6%
0.850	2007 01 04	2018 01 20	147	339	43.4%
Sum			658	321	194.10%
Average			109.67	321	32.35%
Stdev			28.79	0.00	N/A

Table 3-2: Internal structure of GBPUSD D1 timeframe from 2007 01 04 to 2018 01 20 with six different shape ratios.

Instrument	Start	End	Number of Equilibrium Fractal Wave	Number of Peaks and troughs	EFW Index 0.618
EURUSD	2006 09 20	2018 01 20	108	321	33.6%
GBPUSD	2007 01 04	2018 01 20	116	339	34.2%
USDJPY	2008 04 01	2018 01 20	134	326	41.1%
AUDUSD	2008 03 08	2018 01 20	117	333	35.1%
USDCAD	2008 02 19	2018 01 20	120	328	36.6%
NZDUSD	2007 08 15	2018 01 20	122	330	37.0%
EURGBP	2008 05 01	2018 01 20	130	342	38.0%
AUDNZD	2007 08 03	2018 01 20	107	325	32.9%
AUDCAD	2006 08 26	2018 01 20	137	342	40.1%
AUDJPY	2007 04 17	2018 01 20	121	315	38.4%
Average			121.20	330.10	36.7%
Stdev			9.56	8.54	2.60%

Table 3-3: Counting number of equilibrium fractal wave with the shape ratio 0.618 on D1 timeframe for over 3000 candle bars.

Just like Hurst exponent, you can turn the Equilibrium fractal wave index into the technical indicators too. In this case, you can monitor the EFW index over time to check the dominating shape ratio for the financial instrument. Just like the case of Hurst exponent, if you are using too small calculating period, you have the risk of under or over estimating the index values. Therefore, it is important to use the reasonably long calculation period to avoid the risk of under or over estimating the index values.

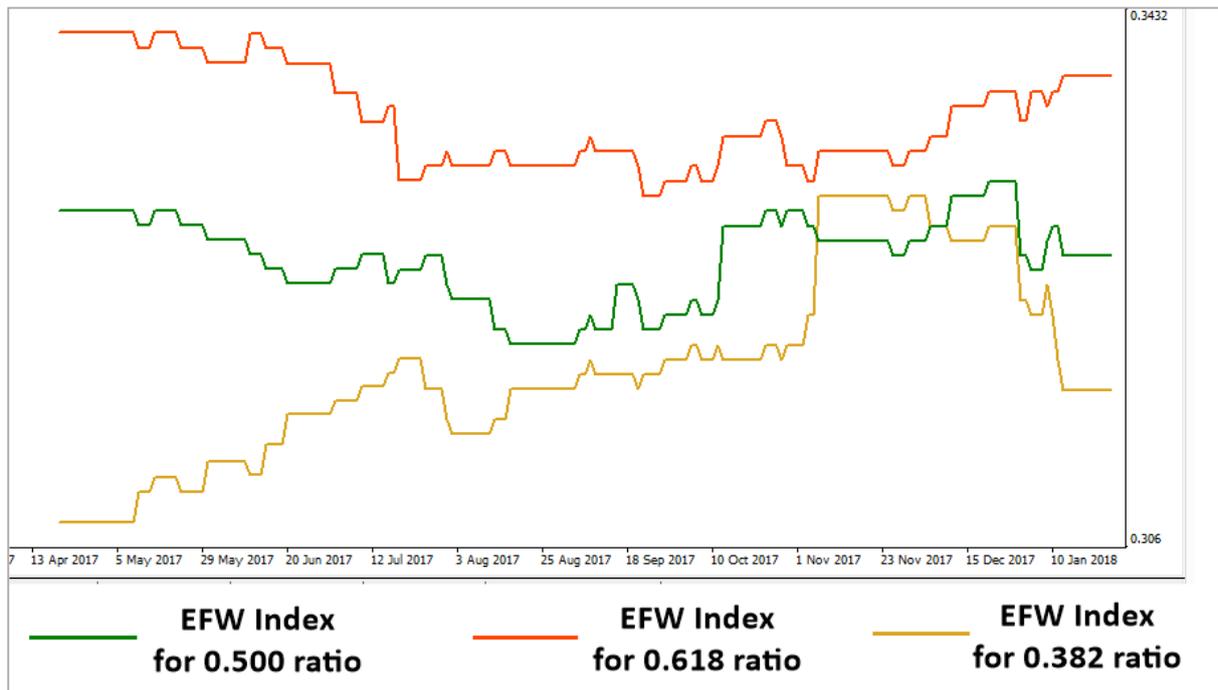


Figure 3-5: EFW index for EURUSD D1 timeframe from 2006 09 20 to 2018 01 20.

#### 4. Practical trading tips with Hurst exponent and Equilibrium fractal wave index

There are many different ways of using Hurst exponent and Equilibrium fractal wave index for the practical trading. In this section, we share one practical tips. In general, Hurst exponent value far away from 0.5 is preferred for your trading because they are more predictable. Based on this knowledge, you can select your best timeframe to trade. For example, in Table 4-1, we can tell that M30 and H4 timeframe is easiest to trade among the six timeframes for EURUSD because they are more predictable than the other timeframes.

	<b>M5</b>	<b>M15</b>	<b>M30</b>	<b>H1</b>	<b>H4</b>	<b>D1</b>
Hurst Exponent	0.553	0.539	0.588	0.58	0.594	0.532

Table 4-1: Hurst exponent for different timeframe for EURUSD.

Likewise, if you are going to trade using the Golden ratio, you can use the Equilibrium fractal wave index to select the best timeframe. For example, in Table 4-2, we can tell that M30 and H1 have more significant EFW index for the shape ratio 0.618. Therefore, it is easier to trade with M30 and H1 using the Golden ratio.

	<b>M5</b>	<b>M15</b>	<b>M30</b>	<b>H1</b>	<b>H4</b>	<b>D1</b>
EFW Index for 0.618	0.284	0.272	0.308	0.300	0.267	0.290

Table 4-2: Equilibrium fractal wave index of the shape ratio 0.618 for different timeframe for EURUSD.

Both Hurst exponent and Equilibrium fractal wave index can be used to select the financial instrument to trade. At the same time, you can use both Hurst exponent and Equilibrium fractal wave index to fine-tune your trading strategy.

## 5. Conclusion

In this article, we have briefly covered the loose self-similarity of the financial market. Hurst exponent can be used to measure the predictability of the financial market. At the same time, Hurst exponent can be used to classify the financial market as either trending or ranging market. With Equilibrium fractal wave index, we can reveal the internal structure of the financial market. With Equilibrium fractal wave index, we can cross compare the internal structure for the different financial instruments. Both Hurst exponent and Equilibrium fractal wave index can be used to select the best timeframe and the financial instrument for your trading. At the same time, you can use these two tools to fine-tuning your trading strategy.